Assignment 2: Vectors and Operators
Due: October 13, 2008

PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 2 : Vector Spaces and Linear Operators

1. Is it a vector space?

Consider a space consisting of three objects:

$$\{|\text{Sun}\rangle, \quad |\text{Moon}\rangle, \quad |\text{Eclipse}\rangle\}$$

which we can abbreviate $|S\rangle$, $|M\rangle$, and $|E\rangle$. Addition of two of them produces,

$$\begin{aligned} |S\rangle + |S\rangle &= |S\rangle, & |S\rangle + |M\rangle &= |E\rangle, \\ |M\rangle + |M\rangle &= |M\rangle, & |S\rangle + |E\rangle &= |S\rangle, \\ |E\rangle + |E\rangle &= |E\rangle, & |M\rangle + |E\rangle &= |M\rangle. \end{aligned}$$

And multiplication by a complex number α results in:

$$\alpha|E\rangle = |E\rangle, \qquad \alpha|S\rangle = \begin{cases} |S\rangle & \text{Re } \alpha > 0 \\ |E\rangle & \text{Re } \alpha = 0 \\ |M\rangle & \text{Re } \alpha < 0 \end{cases}, \qquad \alpha|M\rangle = \begin{cases} |M\rangle & \text{Re } \alpha > 0 \\ |E\rangle & \text{Re } \alpha = 0 \\ |S\rangle & \text{Re } \alpha < 0 \end{cases}.$$

Is this space a linear vector space? Explain your answer.

2. Hermitean and Unitary Operators

Given generic Hermitean operators $\hat{M}$ and $\hat{N}$ and unitary opertors $\hat{U}$ and $\hat{V}$, classify (as Hermitean, anti-Hermitean, unitary, or none of the above) each of the following eight operators:

- $\left(\hat{U}\hat{V}\right)$, $\left(\hat{U}\hat{V}^\dagger\right)$, $\left(\hat{U}+\hat{U}^\dagger\right)$, and $\left(\hat{V}-\hat{V}^\dagger\right)$.
- $\left(\hat{M}\hat{N}\right)$, $\left(\hat{M}+\hat{N}\right)$, $\left(\hat{M}\hat{N}+\hat{N}\hat{M}\right)$, and $\left(\hat{M}\hat{N}-\hat{N}\hat{M}\right)$.

3. Eigenvectors and Eigenvalues

An operator $\hat{M}$ has matrix elements in some orthonormal basis,

$$M_{ij} \;=\; \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$

Is $\hat{M}$ Hermitean? Find the eigenvalues and construct orthonormal eigenkets of $\hat{M}$.